New methods for analyzing multiple processes in muon catalyzed fusion

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Abstract

A package of calculation programs is written to create and analyze the main physical distributions related to the muon catalyzed fusion reactions in the D/T mixture. It involves the simulation of both the muon catalysis processes and registration systems with FADC. Special attention is paid to the absolute calibration problem. It follows from our observation that, in principle, it is not necessary to make special calibration measurements. The programs are intended for use in the large international project TRITON.

1. Introduction

Study of the muon catalyzed fusion (MCF) reactions is now of great interest and is carried out in many laboratories of the world. An extensive experimental program of such investigations is now being carried out in the context of the large international project TRITON aimed to measure the characteristics of the d+t fusion cycle in the double D/T and the triple H/D/T mixture of high density (up to the liquid hydrogen density, LHD). The main parameters to be measured are the absolute neutron yield per muon ($Y_a$), the fusion cycle ($\lambda_c$) and the probability of muon sticking to helium ($\omega$). According to this we shall follow the way of an “effective” analysis where the simplest model reflected in the scheme is considered.

The experimental method is based on the original ideas of the authors of Refs. [1–3]. In the paper [1] it was suggested to measure the time spectrum of the last detected neutron relative to the muon decay. This spectrum is the sum of two exponents [2,3]:

$$\frac{dN}{dt} = \left(\frac{\lambda_0}{\lambda_c}\right) \left[ \omega \lambda_c \exp(-\lambda_0 t) + \epsilon_n \lambda_c (1 - \omega) \exp(-(\lambda_0 + \lambda_c) t) \right].$$

(1)

where $\lambda_0 = 0.455 \mu s^{-1}$ is the muon decay rate, and $\lambda_c$ is defined as $\lambda_c = (\epsilon_n + \omega - \epsilon_n \omega) \lambda_c$, $\epsilon_n$ is the neutron detection efficiency, $\omega$ is the effective sticking probability taking into account the muon sticking in the d+d and t+t accompanied reactions. The ratio of the amplitudes of the slow and fast exponents determines the value of $\omega$: $A_s/A_f = \omega/\epsilon_n(1 - \omega)$.

Another idea [2] is to register the neutron multiplicity (number of detected neutrons $k$, per muon) distribution in some time interval $T$ during which the muon does not disappear. This distribution is a sum of two terms. One of them, a Gaussian (Poisson), $g(k; m)$, with the mean $m = \epsilon_n \lambda_c T$, corresponds to events without sticking and the other one, $f(k)$, is caused by the “sticking” events

$$N(k) = N_1 \left[ f(k) + (1 - \epsilon_n/m)^m g(k; m) \right].$$

(2)

Here $N_1$ is the total number of the first detected neutrons in the time interval $T$ (the sum of $N(k)$). Under the condition $m \gg 1$ the function $f(k)$ is described by the formula $f(k) = y_{k-1} - y_k = y_1 [1 - y_1 (1 - \omega)] [y_1 (1 - \omega)]^{k-1}$, where $y_1$ is the relative yield of the first detected neutrons. It is important that the function $f(k)$ does not depend on $\lambda_c$ for events selected according to the above-mentioned criterion $t_a > T$. As is shown in [2] the analysis of the distributions

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\[ N(k) \] makes it possible to determine the parameters \( \omega/\varepsilon \) and \( \lambda_c \).

Finally, the time distribution of all detected neutrons currently analyzed in the MCF experiments will be measured. They have the well-known form

\[ \frac{dN}{dt} = \varepsilon \lambda_c \exp\left[-(\lambda_0 + \omega \lambda_c) \cdot t\right]. \quad (3) \]

As follows from theory and experiment, the sticking probability should be \( \omega \approx 0.005 \). Under optimal D/T conditions (high density and high tritium concentration) the value of the cycling rate can achieve \( 100-150 \mu s^{-1} \). A large “full absorption” neutron detector (ND) \[4\] is used to register 14 MeV neutrons from the d+t fusion reaction. It consists of two identical parts (11 1 each) symmetrically located around the target. Its detection efficiency is \( 2 \times (10^{-15})\% \).

The “physical” distributions (1)–(3) have been considered in Refs. \[1-3\]. However, in the real experimental conditions, where the value of the measured neutron yield can be \( Y_{\text{exp}} \approx 20-30 \mu s \), they can be substantially distorted due to pile up effects. To discriminate these distortions one decides to measure the ND charge spectra \[3\] instead of the currently measured distributions of the number of events. Flash ADCs \( 256 \) (amplitude) \( \times \) \( 2048 \) (time) with a strobe pulse frequency of 100 MHz are used for this purpose. But some distortions can remain since the time interval between strobe pulses is approximately the same as the ND signal duration \[5\]. Moreover, distortions would be caused by a threshold in the amplitude measurements introduced to discriminate the background. The most serious problem is to provide the correct charge calibration which is important for the absolute yield measurement.

To deal with these problems we have developed a package of Monte-Carlo programs simulating measurements with a FADC. Special attention was paid to the charge calibration procedure. Using these programs we tried to estimate the possible distortion in the measured spectra as depending on the detector signal parameters, neutron multiplicity and threshold. The simulated spectra were fitted with formulae (1)–(3) for this purpose. Finally, these programs are shown to be very useful for preparing methods of on-line control and analysis. This turns out to be very important due to the extreme difficulty in providing the appropriate conditions (high intensity pulsed source) for testing measurements before the experiment with tritium.

2. Calculation programs

The structure of the calculation package is shown in Fig. 1. The first box is related to simplified kinetics of the MCF process. The current parameters are the deuterium and tritium concentrations, the neutron detection efficiency, \( \omega \) and other characteristics of the process. Each individual muon is under control. The competition between the d+t fusion reaction and the mu-decay is simulated successively from cycle to cycle. The muon sticking to helium, which stops the catalysis, is checked. The time of each neutron from the MCF reaction series is determined in this part.

In the next box (“Detector”) one finds out whether the neutron is detected or not. If yes, then one determines the energy released in the scintillator. The uniform recoil proton energy spectrum was used for this in the simplest case. The neutron detector signal was formed according to the study of its shape in Ref. \[5\]. The possibility of its transformation was provided. An example of the detector signal series (the first 400 ns) caused by one muon is given in Fig. 2.

The third block imitates a FADC. Here the detector signals are scanned with a strobe pulse series with a random phase. We will call the amplitude codes obtained as the amplitudes and their sum the charge. Different amplitude thresholds were used in the calculations. A series of successive codes each of which is above the threshold will be called the cluster and their sum the cluster charge.

The next two blocks of the programs are designed to “pack” (and “unpack”) data. These programs were necessary to devise on-line control methods and check them before the experiment. Finally, in the last blocks different distributions were accumulated and analyzed.

The main programs were tested in the “event mode”, where the calculated distributions should obey the
formulae (1)–(3). For this purpose the ND detector signal was accepted as a delta function (in time) with a charge equal to unity. In the “charge mode” (with a FADC) the detector signal was described as

$$s_0(t) = t \cdot \exp[-t/\tau], \quad \tau = 10\,\text{ns}, \quad (4)$$

and broken off at $\Delta t = 30$ or $50\,\text{ns}$. Some special shapes (for instance, a triangle) were also considered.

After all calculations it was decided to shorten the “original” ND signal (4) to provide more correct time measurements. It turned out that the most simple way was to do this as follows:

$$s(t) = s_0(t) \quad \text{for } t = 0 \div \tau,$$

$$s(t) = s_0(\tau) - \left[1 - (t - \tau)/\Delta t\right] \quad \text{for } t = \tau \div \tau + \Delta t \quad (5)$$

with $\tau = 10\,\text{ns}$ and $\Delta t = 20\,\text{ns}$. Therefore, we repeated the main calculations for this signal shape. Fig. 3 shows the shape of the ND signal. The dashed curve corresponds to Eq. (4) and the solid one to Eq. (5). Circles are the measurements for the transformed signal. As is seen, approximation function (5) describes the real signal well.

In most cases, the values

$$ \varepsilon = 0.3, \quad \lambda_c = 100\,\mu\text{s}^{-1}, \quad \omega = 0.005 \quad (6)$$

were used. Of course, they correspond to an extreme case (really, as expected, $\varepsilon \lambda_c \leq 15\,\mu\text{s}^{-1}$), but it allows an estimate of the most possible systematic distortions due to pile up effects.

3. Results

Let us first consider “usual” time distributions of all detected neutrons. The use of the “single mode”, when only the first code (above the threshold) in the cluster is considered, results in substantial violation of the time spectrum shape. Obviously, in the limit of very high neutron multiplicity events will be accumulated only in the first channel (channels) of this spectrum. Thus, we must use all codes.

This superfluous information leads to some spectrum distortions of the type of differential nonlinearity, which mostly manifests itself for low statistics. Besides, the problem of correct determination of uncertainty in the spectrum parameters found from the standard fit procedure requires special consideration.

In the present investigation, the main aim is to estimate possible systematic errors in the spectrum slope values. In Fig. 4 time distributions of all detected neutrons are presented for high ($30\,\text{n}/\mu$) and low ($6\,\text{n}/\mu$) neutron multiplicity. The lower spectrum corresponds to values (6) and statistics $N_0 = 10^4$, the upper spectrum was calculated with $\varepsilon \lambda_c = 3\,\mu\text{s}^{-1}$ and $N_0 = 8 \times 10^4$. Both distributions were normalized to the average “unit charge” $q = Q/N_0$, where $Q$ is the total charge and $N_0$ is the total number of detected neutrons, and fitted with expression (3), the finite time resolution being taken into account [6]. The fitting functions are shown in the figure by curves.
As can be seen from the figure, some distortions show up in this rather low statistics. The optimal values of the exponent slope were found to be

\[ 0.964(3) \, \mu s^{-1} \text{ (expected value 0.955 \, \mu s^{-1})} \]

and

\[ 0.508(3) \, \mu s^{-1} \text{ (0.505 \, \mu s^{-1})} \]

for the high and low cycling rate, respectively. Thus, the systematic shift in the exponent slope does not exceed 1%. To exclude possible influence of the fact that the ND signal duration is comparable with the strobe pulse interval we check our fit with the distributions summarized over four channels. After this the value of \( \chi^2 \) becomes close to the expected one, that provides the correct determination of the parameter uncertainties found in the fit.

Experimentally, the charge calibration procedure can be done in exposures with low neutron multiplicity where pile up effects are poor and the neutron number can be reliably determined. The question is whether this calibration is preserved for exposures with high neutron multiplicity, especially if one uses a rather high threshold. To check it, we carried out calculations for high (30 n/\mu) and low (5 n/\mu) multiplicity at different threshold values. The results are presented in Fig. 5.

The relative change in the unit charge

\[ \delta q = \left[ Q/N_n (\varepsilon \lambda c = 20 \, \mu s^{-1}) \right] / \left[ Q/N_n (\varepsilon \lambda c = 2 \, \mu s^{-1}) \right] - 1 \]

is plotted in it as a function of the relative (to the maximum signal amplitude in the proton recoil spectrum) threshold. As is seen from the figure, the change in the calibration value is only 1–2% for the relative threshold \( \simeq 10\% \).

Another method of charge calibration follows from consideration of the neutron multiplicity distribution (2). The parameter \( m \) is the mean value of the Gaussian \( m = \varepsilon \lambda c T \) and at the same time it determines the relative part of the Gaussian component \((1 - \omega/\varepsilon)\omega\). This fact is well manifested in Fig. 6, where multiplicity distributions are presented for three values of \( T \): 1 \, \mu s (a), 2 \, \mu s (b) and 4 \, \mu s (c). One can see that as the position of the peak changes, its relative part also changes.

While the charge distribution is analyzed with formula (2) the mean of the Gaussian should be multiplied by the unit charge: \( M = m q = O/n \). But the relative part of the Gaussian does not depend on the charge and is determined by the “physical” quantity \( m \). Thus, fit of the measured distribution with independent optimization of the parameters \( M \) and \( m \) makes it possible to obtain \( q \).

Thus, the main characteristics of the d-t fusion cycle \( \lambda_c \), \( \omega \) and \( \varphi = \varepsilon \lambda c/(\lambda_0 + \omega \lambda c) \) can be determined, in principle, from the analysis of the multiplicity distribution.

Fig. 7 shows the multiplicity distributions simulated for \( T = 2 \, \mu s \) with values (6) of \( \lambda_c \), \( \varepsilon \) and \( \omega \). The dashed curve corresponds to the “event mode”. The spectrum obtained in the “charge mode” is shown by circles. It is scaled according

![Fig. 6. Neutron multiplicity distributions accumulated during the time interval \( T = 1 \, \mu s \) (a), \( T = 2 \, \mu s \) (b) and \( T = 4 \, \mu s \) (c).](image1)

![Fig. 7. Neutron multiplicity distributions simulated for \( T = 2 \, \mu s \) with \( \lambda_c \), \( \varepsilon \) and \( \omega \) values (6) in the “charge mode” (circles) and “event mode” (dashed curve). The solid curve corresponds to formula (2) with the optimal parameters found from the fit.](image2)
to the calculated value of the unit charge. The solid curve is function (2) with the optimal parameters found from the fit.

The expected values of those parameters are

\[ m = 60 \quad \text{and} \quad \omega/e = 0.0167. \]

The values found from the analysis of the “physical distribution” turned out to be

\[ m = 59.7 \pm 0.2, \quad \omega/e = 0.0171 \pm 0.02 \]

which are very close to the expected ones.

If one independently varies the parameters \( m \) and \( M = mq \) (the expected value for the normalized spectrum is \( q = 1! \)), one obtains

\[ M = 59.6 \pm 0.2, \quad \omega/e = 0.0179 \pm 0.008, \quad m = 54.1 \pm 4.6. \]

As is seen, the accuracy changes for the worse, especially for the value of \( m \). This is due to correlation between the two parameters (\( m \) and \( \omega/e \)) determining the relative part of the unsticking (sticking) events. (As follows from the further study, expansion of the observation interval \( T \) allows improvement of the accuracy in \( \xi = \omega/e \) and \( m \). For \( m = 109 \) it is \( \delta t/\xi \approx 2\% \) and \( \delta m/m \approx 4\% \).)

Independent measurement of the unit charge \( q \) can improve the accuracy in the value of \( \omega \). Besides, we can also use the value of \( \varepsilon \lambda c \) found from the analysis of the “\( t_c - t_n \)” distribution and thus to derive the value of \( q \) from the mean of the Gaussian: \( m = \varepsilon \lambda c T \).

Accurate measurements of the “\( t_c - t_n \)” distribution with the FADC is the subject of intensive discussions among the members of the MCF community because the expected value of the fast exponent slope \( \varepsilon \lambda c \) in expression (1) is of the order of the ND signal duration and the strobe pulse interval. To study the problem we calculated this spectrum for various conditions. An example of the spectrum obtained for values (6) and the ND signal duration 30 ns is shown in Fig. 8 (circles). To accumulate it we use the time difference between the mu-decay electron and the last FADC code above the threshold.

The simulated distributions were fitted with expression (1) with account of the finite time resolution factor. Both this resolution and the time zero (\( t_0 \)) position were found from the fit. The expected values of the main parameters are the fast exponent slope \( \lambda_f = \lambda_0 + \varepsilon \lambda c = 30.5 \mu s^{-1} \) and the ratio between the slow and fast component amplitudes \( A_s/A_f = \omega/e = 0.0167 \). The results obtained are placed in Table 1.

To check the kinetic simulation and the fit procedure, the results were firstly obtained for the “event mode”. They are presented in the third column of the Table 1. Agreement

![Fig. 8. Time spectrum of the last detected neutrons relative to the muon decay electron (points). The curve is function (1) with the optimal parameters found from the fit.](image)

![Fig. 9. FADC amplitude spectrum calculated for ND signals of shape (4) uniformly distributed in amplitude. The solid histogram corresponds to high neutron multiplicity 30 n/\mu and the dashed one to 3 n/\mu.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expected value</th>
<th>Fit for the “event mode”</th>
<th>Fit for the “charge mode”</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_s/A_f )</td>
<td>0.0167</td>
<td>0.0171(3)</td>
<td>0.0173(2)</td>
</tr>
<tr>
<td>( \lambda_f, \mu s^{-1} )</td>
<td>30.8</td>
<td>30.6(3)</td>
<td>29.5(2)</td>
</tr>
</tbody>
</table>

Table 1
Parameters of the “\( t_c - t_n \)”-distribution
with the expected values is within 1.2%. In the next three columns we show the parameters found from the analysis of distributions simulated for real conditions with the FADC. As is seen from the table, the use of the FADC results in some systematic shifts in the values of parameters, most clearly manifesting themselves for long ND signals.

Another major problem is related to the employment of the detection efficiency calculations. Currently, to check them and to determine the efficiency loss due to threshold, we reconciled the calculated response function of the recoil proton energy with the measured one. In the present case, to eliminate the distortions due to pile up effects this procedure must be done with the charge distributions obtained at low neutron multiplicity. But some systematic distortions due to the threshold can persist even in this extreme case.

To study the problem, we have calculated the amplitude and charge distributions for different conditions. The original response function was taken to be uniformly distributed from zero to some maximum value or put as some fixed value. Fig. 9 shows the FADC amplitude distribution simulated for the ND signals of “usual” shape (4) uniformly distributed in amplitude. As expected, reproduction of the original spectrum is unsatisfactory in this case.

The charge measurements (sum of the amplitude codes inside the cluster) should reproduce the original spectrum much more correctly. First of all, to check the programs, we considered the ND signal shape as a triangle with the base equal to the double strobe pulse interval (20 ns). In this ideal case the charge value of a separate signal should not depend on the phase of the strobe pulse series. The appropriate spectra are shown in Fig. 10 for low (2 n/µ) and high (20 n/µ) neutron multiplicity. As is seen from the figure, for low multiplicity the spectra obtained with the FADC reproduce the initial ones rather well.

At the next stage we considered the charge distributions for the real shape of the ND signal. Since possible distortions are expected to be as large as the ND signal is short, we present the results for the shortened signal of shape (5). The calculations were made for the uniform amplitude distribution with and without the amplitude threshold. Solid histograms in Fig. 11 represent the distribution obtained for the minimal threshold equal to 2% of the maximum ND signal amplitude. Its comparison with the appropriate spectrum of Fig. 10(c) shows that measurements with the real signal lead to some deterioration of the charge resolution under satisfactory reproduction of the spectrum shape, though.

From the physical point of view, the charge (energy) threshold should be introduced to eliminate events from the accompanying d–d fusion reaction (the maximum recoil

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**Fig. 10.** Charge distributions obtained for a triangular signal. The upper spectra correspond to the constant amplitude and the lower ones to the uniformly distributed one. The left pictures (a, c) are related to low neutron multiplicity (2 n/µ) and the right ones (b, d) to high multiplicity (20 n/µ).

**Fig. 11.** Charge spectra calculated for the ND signals of shape (5) with the uniformly distributed amplitude. Solid histograms are obtained with a low threshold and the dashed ones correspond to a 20% threshold relative to the maximum amplitude value. Case (a) corresponds to the amplitude restriction inside the cluster and variant (b) to the “usual” amplitude threshold.
proton energy is equal to 0.7 MeV of an equivalent electron energy, $E_{ee}$) and low-energy background. Note that the appropriate energy for the d-t fusion reaction to be studied is $E_{ee} \approx 7$ MeV. As follows from our consideration, the use of the "usual" amplitude limitation leads to substantial distortions of the final charge distributions (see, Fig. 11(b)). Therefore we used the following algorithm. Producing charge spectra we controlled the maximum value of the amplitude inside the cluster and considered only those cluster charges for which this amplitude exceeded the chosen threshold value. As is seen from Fig. 11(a), this provides conservation of the charge spectrum shape except for low channels.

Fig. 12 shows the results of the direct experimental check of our charge measurement methods. The spectra shown in it were obtained with the $^{60}$Co γ-source. The histogram represents the measurement with the FADC and circles are the same for a "usual" charge digitizer (QCD). As can be seen from the figure, there is very good agreement between both spectra with exception of the lowest charge values (in this case it is due to the threshold effect in the measurements with the QCD).

4. Conclusion

The μCF experiment has been modelled, starting with the kinetics of the process and ending with the analysis of the data obtained. The results give us an assurance that we are now able to effectively analyze the data to be measured in the planned experiment on determination of the main MCF parameters in the liquid D/T mixture. The developed method allows estimation of uncertainties in the parameters for real experimental conditions. It follows from our consideration that the neutron multiplicity $\gamma$, cycling rate $\lambda$, and sticking probability $\omega$ can be determined with an accuracy of few percent.

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References